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Static and dynamic critical properties of 3D systems with long-range correlated quenched defects

V V Prudnikov, P V Prudnikov and A A Fedorenko

Department of Theoretical Physics, Omsk State University 55a, Pr Mira, 644077, Omsk, Russia

E-mail: prudnikv@univer.omsk.su

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Abstract. A field-theoretic description of static and purely relaxational dynamic critical behaviour of systems with quenched defects obeying power law correlations $\sim |x|^{-a}$ for large separations x is given. Directly, for three-dimensional systems and for different values of the correlation parameter, $2 \leqslant a \leqslant 3$, a renormalization analysis of the scaling functions in the two-loop approximation is carried out, and the fixed points corresponding to the stability of various types of critical behaviour are identified. The obtained results essentially differ from results evaluated by a double ε, δ-expansion. The static and dynamic critical exponents in the two-loop approximation are calculated with the use of the Padé–Borel summation technique.

1. Introduction

In recent years, much effort has been devoted to investigating the critical behaviour of solids containing quenched defects. In most papers considerations have been restricted to the case of point defects with small concentrations so that the defects and corresponding random fields have been assumed to be Gaussian distributed and δ-correlated.

For the first time, in the work of Weinrib and Halperin (WH) [1], we have been offered a model of the critical behaviour of a disordered system in which the correlation function of the random local transition temperature $g(x-y) = \langle\langle T_c(x)T_c(y)\rangle\rangle - \langle\langle T_c(x)\rangle\rangle^2$ falls off with distance as a power law $\sim |x-y|^{-a}$. They showed that for $a \geqslant d$ long-range (LR) correlations are irrelevant and the usual short-range (SR) Harris criterion [2] $2-d\nu_0 = \alpha_0 > 0$ of influence of δ-correlated point defects is realized, where d is the spatial dimension, and ν_0 and α_0 are the correlation-length and the specific-heat exponents of the pure system. For $a < d$ the extended criterion $2 - a\nu_0 > 0$ of the influence of disorder on the critical behaviour was established. As a result, a wider class of disordered systems, not only the three-dimensional (3D) Ising model with δ-correlated point defects, can be characterized by a new type of critical behaviour. So, for $a < d$ a new LR disorder stable fixed point (FP) of the renormalization group recursion relations for systems with a number of components of the order parameter $m \geqslant 2$ was discovered. The critical exponents were calculated in the one-loop approximation using a double expansion in $\varepsilon = 4-d \ll 1$ and $\delta = 4-a \ll 1$. In the case $m = 1$ the accidental degeneracy of the recursion relations in the one-loop approximation did not permit them to find LR disorder stable FP, but a change in critical behaviour of the model from the SR to the LR-correlation type was predicted for $\delta > \delta_c = 2(6\varepsilon/53)^{1/2}$. Korzhenevskii *et al* [3] proved the existence of the LR disorder stable FP for the one-component WH model and also found characteristics of this type of critical behaviour. Also, they considered a very interesting

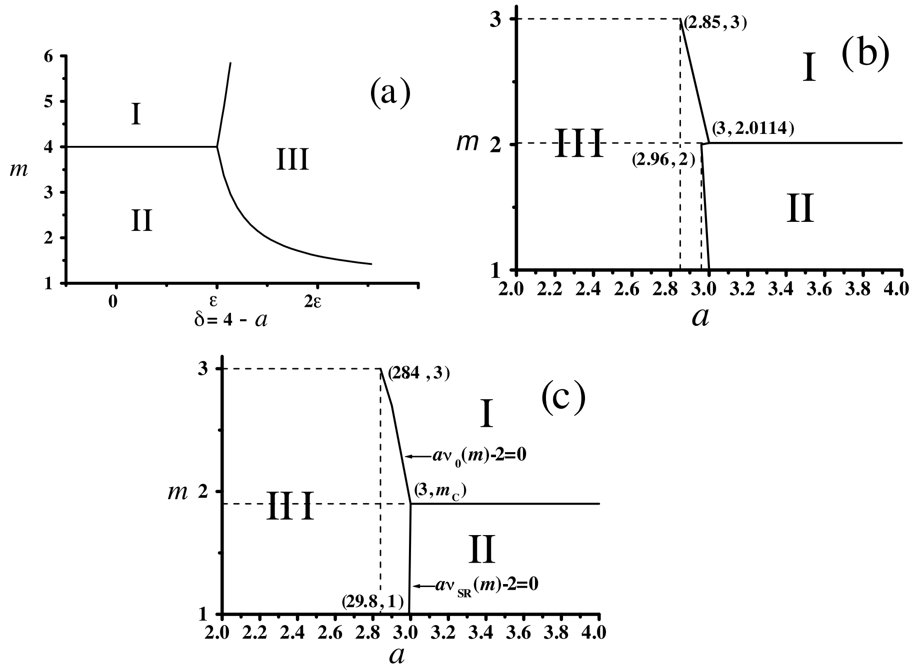

Figure 1. Regions of the various types of critical behaviour which have been determined: (*a*) in [1] on the basis of the double ε, δ-expansion; (*b*) in this paper with use of the field-theoretic description in a two-loop approximation for the 3D WH model; (*c*) in this paper taking into consideration the higher orders of approximation.

model of the critical behaviour of crystals with LR correlations caused by point defects with degenerate internal degrees of freedom [3, 4].

The models with LR-correlated quenched defects have both theoretical interest due to the possibility of predicting new types of critical behaviour in disordered systems and experimental interest due to the possibility of realizing RL-correlated defects in disordered solids containing fractal-like defects [3]. However, numerous investigations of pure and disordered systems performed with the use of the field-theoretic approach show that the predictions made in the one-loop approximation, especially on the basis of the ε-expansion, can differ strongly from the real critical behaviour [5–8]. Therefore, the map of regions with the various types of critical behaviour derived for the WH model on the basis of ε, δ-expansion [1] (figure 1(*a*)) may not correspond to the critical behaviour of the 3D WH model for different values of m and a. In this case the results for the models with LR-correlated defects derived with the use of ε, δ-expansion [1, 3, 4, 9–11] must be corrected. To shed light on this question and to determine more accurately the dependence of the critical behaviour on the number of components of the order parameter m and the values of correlation parameter a, we have constructed a field-theoretical description of the 3D WH model in the two-loop approximation for the values of a in the range $2 \leqslant a \leqslant 3$. For dynamics, we concentrate on a purely relaxational model with no conserved quantities (model A in the classification of Hohenberg and Halperin [12]).

In section 2 we describe a Lagrangian theory of critical dynamics of the WH model with LR-correlated defects and use the replica method to construct the generating functional for dynamic correlation and response functions. Renormalization of the model is discussed in section 3. Scaling β functions and the FPs corresponding to the stability of various types of critical behaviour are determined in section 4. The calculation of the critical exponents with the use of the Padé–Borel summation method and discussion of the main results are given in section 5.

2. The model and generating functional

We consider an O(m)-symmetric Ginzburg–Landau–Wilson model of a disordered system defined by the effective Hamiltonian

$$\mathcal{H}(\phi, V) = \int \mathrm{d}^d x \left[\frac{1}{2} \sum_{\beta=1}^{m} [r_0(\phi^\beta)^2 + |\nabla\phi^\beta|^2 + V(x)(\phi^\beta)^2] + \frac{u_0}{4!} \left(\sum_{\beta=1}^{m} (\phi^\beta)^2 \right)^2 \right] \tag{2.1}$$

where $\phi(x, t)$ is the m-component order parameter and $V(x)$ is the potential of defects. As usual, r_0 is taken to be linear in temperature and u_0 to be a positive constant. The concentration of defects is taken to be well below the percolation limit. The average of V over the quenched random distribution is taken to be zero (otherwise its constant average value could be absorbed into r_0) and according to the WH model the second moment of the distribution has the form $\langle\langle V(x)V(y)\rangle\rangle = 8g(x - y) \sim |x - y|^{-a}$.

The dynamical behaviour of the system in the relaxation regime near the critical temperature can be described by the Langevin equation for the order parameter [12]

$$\frac{\partial \phi^\beta(x, t)}{\partial t} = -\lambda_0 \frac{\delta \mathcal{H}}{\delta \phi^\beta(x, t)} + \eta^\beta(x, t) \tag{2.2}$$

where λ_0 is the Onsager kinetic coefficient. The Gaussian random-noise source $\eta(x, t)$ has the probability functional

$$P_\eta(\eta) = A_\eta \exp\left[-(4\lambda_0)^{-1} \int \mathrm{d}^d x \, \mathrm{d}t \, \eta^\beta(x, t)\eta^\beta(x, t) \right] \tag{2.3}$$

where a summation over $\beta = 1, \ldots, m$ is understood. This functional may conveniently be rewritten using auxiliary response fields $\tilde{\phi}^\beta(x, t)$ [13]. For later convenience, we introduce the source $\tilde{J}^\beta(x, t)$

$$P_\eta(\eta, \tilde{J}) = A_\eta \int \mathcal{D}\tilde{\phi} \exp\left[-\int \mathrm{d}^d x \, \mathrm{d}t \tilde{\phi}^\beta (\lambda_0^{-1} \tilde{\phi}^\beta + \mathrm{i}\lambda_0^{-1} \eta^\beta - \tilde{J}^\beta) \right]. \tag{2.4}$$

In accordance with [14, 15], dynamic correlation and response functions may be obtained from the generating functional

$$G(J, \tilde{J}) = -\ln W(J, \tilde{J}) \tag{2.5}$$

where

$$\begin{aligned} W(J, \tilde{J}) &= \int \mathcal{D}\eta P_\eta(\eta, \tilde{J}) \exp\left(\int \mathrm{d}^d x \, \mathrm{d}t \, J^\beta \phi^\beta \right) \\ &= \int \mathcal{D}\phi \mathcal{D}\tilde{\phi} \det \left| \frac{\partial \eta}{\partial \phi} \right| \exp\left(\int \mathrm{d}^d x \, \mathrm{d}t \, (J^\beta \phi^\beta + \tilde{J}^\beta \tilde{\phi}^\beta) - \mathcal{L} \right). \end{aligned} \tag{2.6}$$

Here, η is to be expressed in terms of ϕ by substitution from (2.2),which yields the Lagrangian

$$\mathcal{L} = \int \mathrm{d}^d x \, \mathrm{d}t \left[\tilde{\phi}^\beta \lambda_0^{-1} \tilde{\phi}^\beta + \mathrm{i}\tilde{\phi}^\beta \left(\lambda_0^{-1} \frac{\partial \varphi^\beta}{\partial t} + \frac{\delta \mathcal{H}}{\delta \phi^\beta} \right) \right]. \tag{2.7}$$

The effect of the Jacobian in (2.6) may be implemented perturbatively by simply omitting single response propagator loops [16] and we drop it hereafter.

Green functions generated by $G(J, \tilde{J})$ are to be further averaged over the random potential $V(x)$. This averaging is most efficiently carried out by means of the replica method (although direct term-by-term averaging generates precisely the same perturbation series). In the usual way, the formal identity

$$\ln W = \lim_{n\to 0}\left\langle\left\langle \frac{W^n - 1}{n} \right\rangle\right\rangle$$

(where double angle brackets denote the defect average over the probability distribution $P(V)$) leads us to study the generating functional

$$W^{(n)} = \int \prod_{i=1}^{n} \mathcal{D}\phi_i \mathcal{D}\tilde{\phi}_i \left\langle\left\langle \exp\left[-\sum_{j=1}^{n} \left(\mathcal{L}(\phi_j \tilde{\phi}_j) - \int \mathrm{d}^d x\, \mathrm{d}t\, (J_j^\beta \phi_j^\beta + \tilde{J}_j^\beta \tilde{\phi}_j^\beta) \right) \right] \right\rangle\right\rangle$$
$$= \int \prod_{i=1}^{n} \mathcal{D}\phi_i \mathcal{D}\tilde{\phi}_i \exp\left(-\mathcal{L}^{(n)} + \sum_{j=1}^{n} \int \mathrm{d}^d x\, \mathrm{d}t\, (J_j^\beta \phi_j^\beta + \tilde{J}_j^\beta \tilde{\phi}_j^\beta) \right). \tag{2.8}$$

To obtain the replicated Lagrangian $\mathcal{L}^{(n)}$, we need to compute the average

$$\left\langle\left\langle \exp\left(-\mathrm{i} \int \mathrm{d}^d x\, \mathrm{d}t\, V(x) \tilde{\phi}_i^\beta \phi_i^\beta \right) \right\rangle\right\rangle = \int \mathcal{D}V\, P(V) \exp\left(-\mathrm{i} \int \mathrm{d}^d x\, \mathrm{d}t\, V(x)\, \tilde{\phi}_i^\beta \phi_i^\beta \right)$$
$$\sim \exp\left(-4 \int \mathrm{d}^d x\, \mathrm{d}^d y\, \mathrm{d}t\, \mathrm{d}t'\, g(x-y)\, \tilde{\phi}_i^\beta(x,t) \phi_i^\beta(x,t) \tilde{\phi}_j^\gamma(y,t') \phi_j^\gamma(y,t') \right). \tag{2.9}$$

Substituting in (2.8), we obtain

$$\mathcal{L}^{(n)} = \sum_i \int \mathrm{d}^d x\, \mathrm{d}t \left[\lambda_0^{-1} \tilde{\phi}_i^\beta \tilde{\phi}_i^\beta + \mathrm{i}\tilde{\phi}_i^\beta \left(\lambda_0^{-1} \frac{\partial \phi_i^\beta}{\partial t} - \nabla^2 \phi_i^\beta + r_0 \phi_i^\beta \right) + \frac{\mathrm{i}}{3!} u_0 \tilde{\phi}_i^\beta \phi_i^\beta \phi_i^\gamma \phi_i^\gamma \right]$$
$$+4 \sum_{ij} \int \mathrm{d}^d x\, \mathrm{d}^d y\, \mathrm{d}t\, \mathrm{d}t'\, g(x-y) \tilde{\phi}_i^\beta(x,t) \phi_i^\beta(x,t) \tilde{\phi}_j^\gamma(y,t') \phi_j^\gamma(y,t'). \tag{2.10}$$

The properties of the original disordered system are obtained in the replica number limit $n \to 0$. The average generating functional is now given by

$$\tilde{G}(J, \tilde{J}) = \langle\langle G(J, \tilde{J}) \rangle\rangle = -\lim_{n\to 0} \frac{\ln W^{(n)}(J, \tilde{J})}{n} \tag{2.11}$$

where, on the right-hand side, the sources J_i^β and $\tilde{J}_i^\beta$ are taken to be the same for each replica i. From this average generating functional the connected Green functions $G^{(N,\tilde{N})}$ can be defined by the next expressions:

$$G^{(N,\tilde{N})}_{\{\beta\}_N \{\beta'\}_{\tilde{N}}}(\{x,t\}_N, \{x',t'\}_{\tilde{N}}) = \left\langle \prod_{j=1}^{N} \phi_j^{\beta_j}(x_j, t_j) \prod_{k=1}^{\tilde{N}} \tilde{\phi}_k^{\beta'_k}(x'_k, t'_k) \right\rangle$$
$$= \prod_{j=1}^{N} \frac{\delta}{\delta J^{\beta_j}(x_j, t_j)} \prod_{k=1}^{\tilde{N}} \frac{\delta}{\delta \tilde{J}^{\beta_k}(x'_k, t'_k)} \tilde{G}(J, \tilde{J})|_{J=\tilde{J}=0}. \tag{2.12}$$

It will be convenient to introduce the one-particle irreducible vertex functions $\Gamma^{(N,\tilde{N})}$. Their generating functional is obtained from $\tilde{G}(J, \tilde{J})$ through a Legendre transformation,

$$\Gamma(\phi, \tilde{\phi}) = \tilde{G}(J, \tilde{J}) + \int \mathrm{d}^d x\, \mathrm{d}t\, (J^\beta \phi^\beta + \tilde{J}^\beta \tilde{\phi}^\beta) \tag{2.13}$$

where

$$\phi^\beta = -\frac{\delta \tilde{G}}{\delta J^\beta} \qquad \tilde{\phi}^\beta = -\frac{\delta \tilde{G}}{\delta \tilde{J}^\beta}. \tag{2.14}$$

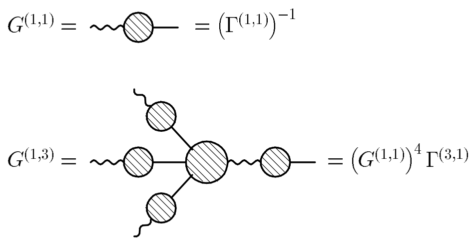

Figure 2. Graphical illustration of the Legendre transform leading to equations (2.17) and (2.18).

Then

$$J^\beta = \frac{\delta\Gamma}{\delta\phi^\beta} \qquad \tilde{J}^\beta = \frac{\delta\Gamma}{\delta\tilde{\phi}^\beta} \tag{2.15}$$

and

$$\Gamma^{(N,\tilde{N})}_{\{\beta\}_N\{\beta'\}_{\tilde{N}}}(\{x,t\}_N, \{x',t'\}_{\tilde{N}}) = \prod_{j=1}^{N} \frac{\delta}{\delta\phi^{\beta_j}(x_j,t_j)} \prod_{k=1}^{\tilde{N}} \frac{\delta}{\delta\tilde{\phi}^{\beta_k}(x'_k,t'_k)} \Gamma(\phi,\tilde{\phi})|_{\phi=\tilde{\phi}=0}. \tag{2.16}$$

The physical significance of the field $\tilde{\phi}$ is easily seen if we add a time-dependent external field $\tilde{J}^\beta(x,t)$ to the right-hand side of the Langevin equation (2.2). This leads directly to the term $\tilde{J}^\beta\tilde{\phi}^\beta$ in equations (2.6) and (2.8). Consequently, the cumulants $G^{(N,\tilde{N})}$ with $\tilde{N} \geqslant 1$ are response functions.

The static correlation functions are obtained (see, e.g. de Dominicis and Peliti [16] and references therein) as the zero-frequency limits of dynamic response functions

$$G^{(N)}_{\text{static}}(q^i) = G^{(1,N-1)}(q^i, \omega^i = 0) \tag{2.17}$$

in the absence of streaming terms from the equation of motion (2.2). Using (2.15) and (2.16) we find that the static vertex functions are given by

$$\Gamma^{(N)}_{\text{static}}(q^i) = \Gamma^{(N-1,1)}(q^i, \omega^i = 0). \tag{2.18}$$

These relations are illustrated for the two- and four-point functions in figure 2.

Also, as generalization of the our dynamical scheme the generating functional for cumulants with insertions of the composite field $\phi^2(x,t)$, which have an independent existence when fluctuations become important, may be introduced:

$$W^{(n)} = \int \prod_{i=1}^{n} \mathcal{D}\phi_i \mathcal{D}\tilde{\phi}_i \exp\left(-\mathcal{L}^{(n)} + \sum_{j=1}^{n} \int \mathrm{d}^d x\, \mathrm{d}t\, (J_j^\beta \phi_j^\beta + \tilde{J}^\beta \tilde{\phi}^\beta + \tfrac{1}{2} I_j \phi_j^2)\right) \tag{2.19}$$

and then the average generating functional is given by

$$\tilde{G}(J,\tilde{J},I) = \langle\langle G(J,\tilde{J},I)\rangle\rangle = -\lim_{n\to 0} \frac{\ln W^{(n)}(J,\tilde{J},I)}{n}. \tag{2.20}$$

The generating functional for vertex functions with insertions is defined through the partial Legendre transformation

$$\Gamma(\phi,\tilde{\phi};I) = \tilde{G}(J,\tilde{J},I) + \int \mathrm{d}^d x\, \mathrm{d}t\, (J^\beta\phi^\beta + \tilde{J}^\beta\tilde{\phi}^\beta) \tag{2.21}$$

which yields

$$\Gamma^{(L,N,\tilde{N})}(\{x,t\}_N, \{x',t'\}_{\tilde{N}}, \{y,\tau\}_L) = \prod_{j=1}^{L} \frac{\delta}{\delta I(y_j,\tau_j)} \Gamma^{(N,\tilde{N})}(\{x,t\}_N, \{x',t'\}_{\tilde{N}}; I)|_{I=0}. \tag{2.22}$$

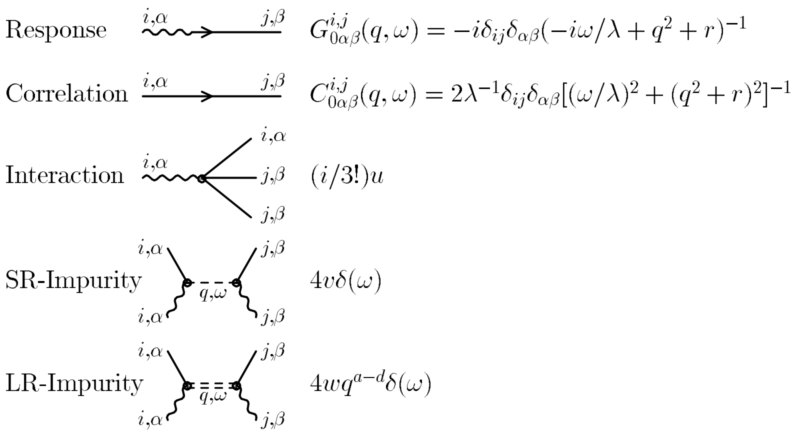

Figure 3. Diagrammatic rules for the perturbation series generating by the Lagrangian (2.10). Momentum and frequency are conserved at each vertex, while the impurity vertices carry the additional constraint shown.

In this case the static vertex functions are given by

$$\Gamma^{(L,N)}_{\text{static}}(q^i) = \Gamma^{(L,N-1,1)}(q^i, \omega^i = 0). \tag{2.23}$$

The Fourier transformation of the interaction vertex $g(x) \sim x^{-a}$ in the replicated Lagrangian (2.10) gives $g(k) = v_0 + w_0 k^{a-d}$ for small k. $g(k)$ must be positive definite, therefore if $a > d$, then the w term is irrelevant, $v_0 \geqslant 0$ and $\mathcal{L}^{(n)}$ (2.10) corresponds to the model with SR-correlated defects [17,18], while if $a < d$, then the w term is dominant at small k and $w_0 \geqslant 0$. After Fourier transformation of the replicated Lagrangian (2.10) on space and time, we arrive at the diagrammatic rules shown in figure 3.

3. Renormalization and renormalization group equation

As is known, in the field-theoretic approach [19] the asymptotic critical behaviour of systems in the fluctuation region are determined by the Callan–Symanzik renormalization group equation for the vertex parts of the irreducible Green functions. To calculate the β functions and the critical exponents as functions of the renormalized interaction vertices u, v and w (scaling γ functions) appearing in the renormalization group equation, we used the method based on the Feynman diagram technique and the renormalization procedure [14, 15, 20].

The Feynman diagrams that contribute to the correlation and response functions involve momentum integrations of dimension d (in our case $d = 3$). Near the critical point the correlation length ξ increases infinitely. When $\xi^{-1} \ll \Lambda$, where Λ is a cutoff in momentum–space integrals (the cutoff Λ serves to specify the basic length scale), the vertex functions are expected to display an asymptotic scaling behaviour for wavenumbers $q \ll \Lambda$. Therefore, one is led to consider the vertex functions in the limit $\Lambda \to \infty$. Since the 'bare' parameters $m_0^2 = r_0 - r_{0c}$ (r_{0c} denotes the critical value of r_0), λ_0, u_0, v_0, w_0 and 'bare' fields $\phi_0, \tilde{\phi}_0$ carry a Λ-dimension, one has to renormalize the theory in order to absorb the divergences of diagrams in a change of parameters and to obtain meaningful expression for the correlation and response functions for $\Lambda \to \infty$.

The required reparametrization employs the next renormalization algorithm developed for Lagrangian field theory. We first define renormalized fields $\phi = Z^{-1/2}\phi_0$ and $\tilde{\phi} = Z^{-1/2}\tilde{\phi}_0$, where now the zero subscripts denote the unrenormalized quantities appearing in section 2.

The relations (2.17) require that both ϕ and $\tilde{\phi}$ are renormalized by the same factor $Z^{-1/2}$. The renormalized composite field can be defined by $\phi^2 = (Z_{\phi^2}/Z)\phi_0^2$. The renormalized vertex functions are given by

$$\Gamma^{(L,N,\tilde{N})}(q^i, \omega^i, m^2, u, v, w, \lambda) = Z^{(N+\tilde{N})/2-L} Z_{\phi^2}^L \Gamma_0^{(L,N,\tilde{N})}(q^i, \omega^i, m_0^2, u_0, v_0, w_0, \lambda_0) \tag{3.1}$$

with renormalized parameters defined by

$$\begin{aligned} m_0^2 &= m^2 Z^{-1} \tilde{m}_0^2(u, v, w, m/\Lambda) \\ u_0 &= m^{4-d} Z^{-2} \tilde{u}_0(u, v, w, m/\Lambda) \\ v_0 &= m^{4-d} Z^{-2} \tilde{v}_0(u, v, w, m/\Lambda) \\ w_0 &= m^{4-a} Z^{-2} \tilde{w}_0(u, v, w, m/\Lambda) \\ \lambda_0^{-1} &= Z_\lambda \lambda^{-1} \end{aligned} \tag{3.2}$$

where $\tilde{m}_0^2$, $\tilde{u}_0$, $\tilde{v}_0$, $\tilde{w}_0$, and all Z-factors are dimensionless functions of renormalized parameters m/Λ, u, v, and w. To determine these dimensionless functions, we require at each order of vertex functions expansion that the renormalized two- and four-point vertex functions contain no divergences for $\Lambda \to \infty$. On dimensional grounds, we then expect that higher-order vertices are also free of divergences. The Z-factors and dimensionless functions $\tilde{m}_0^2$, $\tilde{u}_0$, $\tilde{v}_0$, and $\tilde{w}_0$ are all obtained from normalization conditions for the response function $\Gamma^{(0,1,1)}$, four-point functions $\Gamma^{(0,3,1)}$ and $\Gamma^{(0,2,2)}$ and two-point function $\Gamma^{(1,1,1)}$ with ϕ^2 insertion:

$$\begin{aligned} &\Gamma^{(0,1,1)}(q, -q, \omega; m^2, u, v, w, \lambda)|_{q^2,\omega=0} = m^2 \\ &\frac{\partial}{\partial q^2}\Gamma^{(0,1,1)}(q, -q, \omega; m^2, u, v, w, \lambda)|_{q^2,\omega=0} = 1 \\ &\frac{\partial}{\partial(-i\omega)}\Gamma^{(0,1,1)}(q, -q, \omega; m^2, u, v, w, \lambda)|_{q^2,\omega=0} = \lambda^{-1} \\ &\Gamma^{(0,3,1)}(q^i, \omega^i; m^2, u, v, w, \lambda)|_{q^i,\omega^i=0} = m^{4-d} u \\ &\Gamma_v^{(0,2,2)}(q^i, \omega^i; m^2, u, v, w, \lambda)|_{q^i,\omega^i=0} = m^{4-d} v \\ &\Gamma_w^{(0,2,2)}(q^i, \omega^i; m^2, u, v, w, \lambda)|_{q^i,\omega^i=0} = m^{4-a} w \\ &\Gamma^{(1,1,1)}(q; p, -p; \omega^i; m^2, u, v, w, \lambda)|_{q,p,\omega^i=0} = 1. \end{aligned} \tag{3.3}$$

When we considered a diagrammatic representation of two-point vertex function $\Gamma^{(0,1,1)}$ (figure 4), three types of four-point vertex functions $\Gamma^{(0,3,1)}$, $\Gamma_v^{(0,2,2)}$, and $\Gamma_w^{(0,2,2)}$ and two-point with the ϕ^2 insertion vertex function $\Gamma^{(1,1,1)}$ in the two-loop approximation the diagrams were integrated numerically in $d = 3$ and with values of parameter a determining momentum dependence of the w interaction in the range $2 \leqslant a \leqslant 3$ with changes through the step $\Delta a = 0.01$. Unlike the works using ε, δ-expansion we took into consideration the graphs of the form of figure 5, contributions of which are increased when the values a are removed from $a = 3$.

As is known, the scaling behaviour of vertex functions follows from the Callan–Symanzik renormalization group equations, which can be derived in our case by taking a derivative of equation (3.1) with respect to $\ln m$, at fixed u_0, v_0, w_0, λ_0, and Λ, and have the form

$$\left[m\frac{\partial}{\partial m} + \beta_u\frac{\partial}{\partial u} + \beta_v\frac{\partial}{\partial v} + \beta_w\frac{\partial}{\partial w} + \gamma_\lambda\lambda\frac{\partial}{\partial \lambda} - L(\gamma_{\phi^2} - \gamma_\phi) - \frac{N+\tilde{N}}{2}\gamma_\phi\right]\Gamma^{(L,N,\tilde{N})}(q^i, p^j, \omega^i, \omega^j; m^2, u, v, w, \lambda) = m^2(2-\gamma_\phi)\Gamma^{(L+1,N,\tilde{N})}. \tag{3.4}$$

The right-hand side is asymptotically smaller, as $m/\Lambda \to 0$, (it may be asymptotically neglected at least order by order in perturbation theory) and is assumed not to affect the critical behaviour.

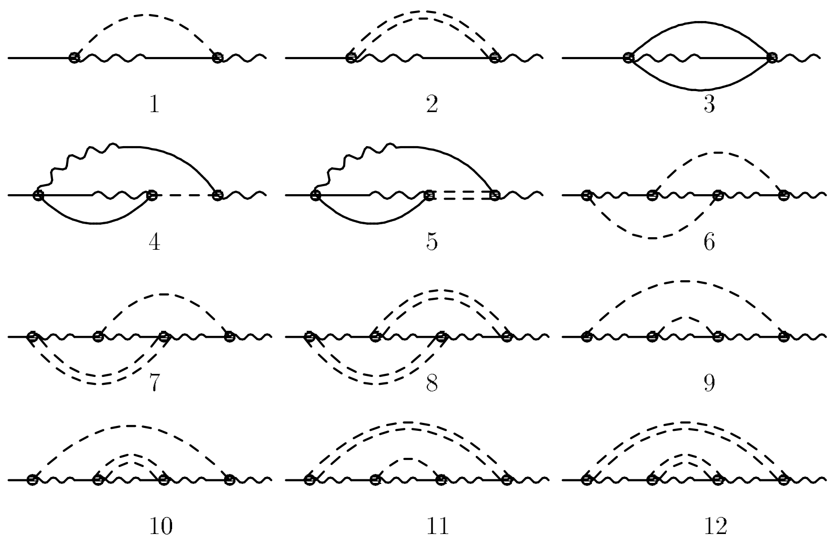

Figure 4. One- and two-loop graphs contributing to the two-point vertex function $\Gamma^{(0,1,1)}(q, \omega)$.

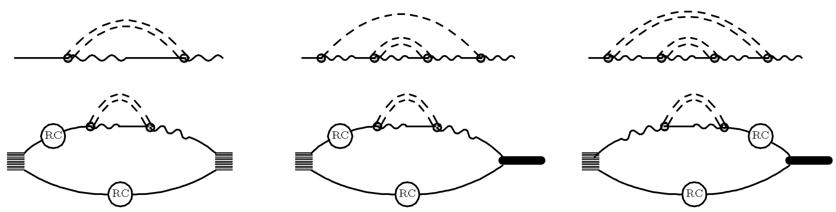

Figure 5. Graphs that, in addition, take into consideration the comparison with other works, using ε, δ-expansion: ■ corresponds to vertices u, v and w; ■■■ corresponds to insertion of the composite field $\phi^2(x, t)$; ——O—— corresponds to dynamic response or correlation functions.

Standard arguments for β and γ functions give the following expressions:

$$
\begin{aligned}
&\beta_u(u, v, w) = \left(m \frac{\partial u}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} &&\beta_v(u, v, w) = \left(m \frac{\partial v}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} \\
&\beta_w(u, v, w) = \left(m \frac{\partial w}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} &&\gamma_\phi(u, v, w) = \left(m \frac{\partial \ln Z}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} \\
&\gamma_{\phi^2}(u, v, w) = \left(m \frac{\partial \ln Z_{\phi^2}}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} &&\gamma_\lambda(u, v, w) = \left(m \frac{\partial \ln Z_\lambda}{\partial m} \right)\bigg|_{u_0, v_0, w_0, \lambda_0} .
\end{aligned}
\tag{3.5}
$$

We computed the Feynman graphs contributing to equations (3.3) and determined Z-factors and functions $u_0(u, v, w)$, $v_0(u, v, w)$, $w_0(u, v, w)$. As a result, we obtained the β and γ functions in the two-loop approximation in the form of the expansion series in renormalized

Table 1. Coefficients for the β functions in equations (3.6).

a	b_1	b_2	b_3	b_4	b_5	b_6
3.01	1.862 631	9.763 843	0.867 400	1.723 696	0.337 829	1.004 987
3.00	1.851 852	9.703 704	0.856 481	1.712 963	0.333 333	1.000 000
2.90	1.751 381	9.149 428	0.761 858	1.614 234	0.297 079	0.961 992
2.80	1.662 830	8.671 819	0.686 998	1.529 491	0.273 802	0.944 294
2.70	1.584 520	8.260 292	0.627 099	1.456 807	0.260 815	0.946 290
2.60	1.515 077	7.906 550	0.578 448	1.394 685	0.256 737	0.968 890
2.50	1.453 357	7.604 029	0.537 918	1.341 947	0.261 215	1.014 801
2.40	1.398 383	7.347 527	0.502 515	1.297 671	0.274 936	1.089 146
2.30	1.349 314	7.132 943	0.468 822	1.261 144	0.299 612	1.200 719
2.20	1.305 402	6.957 111	0.432 135	1.231 831	0.338 622	1.364 436
2.10	1.265 968	6.817 670	0.384 813	1.209 353	0.397 917	1.606 356
2.00	1.230 378	6.713 001	0.312 654	1.193 479	0.488 229	1.974 883

vertices u, v and w. We list here the resulting expansions:

$$\begin{aligned}
\beta_u(u,v,w) &= -u+u^2-\frac{3}{2}uv-(3f_1-f_2)uw-\frac{4(41m+190)}{27(m+8)^2}u^3-\frac{185}{216}uv^2\\
&\quad+\frac{2(25m+131)}{27(m+8)}u^2v+\frac{1}{m+8}(b_1m+b_2)u^2w-b_3uw^2-b_4uvw\\
\beta_v(u,v,w) &= -v-v^2-f_3w^2-(3f_1-f_2)vw+\frac{2(m+2)}{(m+8)}uv-\frac{95}{216}v^3-b_5w^3\\
&\quad+\frac{50(m+2)}{27(m+8)}uv^2+b_6\frac{(m+2)}{(m+8)}uw^2-b_7vw^2-\frac{92(m+2)}{27(m+8)^2}u^2v\\
&\quad-b_8v^2w+b_9\frac{(m+2)}{(m+8)}uvw\\
\beta_w(u,v,w) &= -(4-a)w-(f_1-f_2)w^2-\frac{1}{2}vw+\frac{2(m+2)}{(m+8)}uw+b_{10}w^3-b_{11}vw^2\\
&\quad-\frac{23}{216}v^2w-\frac{92(m+2)}{27(m+8)^2}u^2w+b_{12}\frac{(m+2)}{(m+8)}uw^2+\frac{23(m+2)}{27(m+8)}uvw\\
\gamma_\phi(u,v,w) &= \frac{1}{2}f_2w+\frac{8(m+2)}{27(m+8)^2}u^2+\frac{1}{108}v^2+c_1w^2-\frac{2(m+2)}{27(m+8)}uv\\
&\quad-c_2\frac{(m+2)}{(m+8)}uw+\frac{1}{4}c_2vw\\
\gamma_{\phi^2}(u,v,w) &= -\frac{m+2}{m+8}u+\frac{1}{4}v+\frac{1}{2}f_1w+\frac{2(m+2)}{(m+8)^2}u^2+\frac{1}{16}v^2+c_3w^2+\frac{1}{4}c_4vw\\
&\quad-\frac{(m+2)}{2(m+8)}uv-c_4\frac{(m+2)}{(m+8)}uw\\
\gamma_\lambda(u,v,w) &= \frac{1}{4}v+\frac{1}{2}(f_1-f_2)w+0.226777\frac{(m+2)}{(m+8)^2}u^2+\frac{23}{432}v^2+c_5w^2\\
&\quad+c_6vw+c_7\frac{(m+2)}{(m+8)}uw-\frac{5(m+2)}{54(m+8)}uv\\
f_1 &= \frac{(a-2)(a-4)}{2\sin(\pi a/2)} \qquad f_2=\frac{(a-2)(a-3)(a-4)}{48\pi\sin(\pi(a/2-1))} \qquad f_3=\frac{(2a-5)(2a-7)}{2\sin(\pi(a-3/2))}
\end{aligned} \tag{3.6}$$

where the coefficients b_i and c_i for different values of parameter a in the range $2 \leqslant a \leqslant 3$ are given in tables 1–3.

The series (3.6) are normalized by a standard change of variables [6,7] $u \to 6u/(m+8)J$, $v \to v/32J$, $w \to w/32J$, so that the coefficients of the terms u, u^2 and v, v^2 in β_u and β_v become 1 in modulus, where $J = \int d^dq/(q^2+1)^2$ is the one-loop integral.

Table 2. Coefficients for the β functions in equations (3.6) (a continuation of table 1).

a	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
3.01	1.118 646	1.220 480	2.869 061	−0.109 703	0.216 033	0.864 131
3.00	1.106 481	1.212 963	2.851 852	−0.106 481	0.212 963	0.851 852
2.90	1.002 356	1.143 678	2.692 492	−0.078 756	0.183 943	0.735 771
2.80	0.923 071	1.083 977	2.553 857	−0.055 369	0.157 468	0.629 873
2.70	0.863 671	1.032 536	2.433 062	−0.034 759	0.132 927	0.531 707
2.60	0.820 670	0.988 319	2.327 810	−0.015 651	0.109 805	0.439 222
2.50	0.791 618	0.950 504	2.236 243	0.003 123	0.087 655	0.350 621
2.40	0.774 801	0.918 441	2.156 843	0.022 783	0.066 066	0.264 264
2.30	0.769 001	0.891 618	2.088 367	0.044 810	0.044 645	0.178 581
2.20	0.773 244	0.869 639	2.029 786	0.071 264	0.023 001	0.092 005
2.10	0.786 402	0.852 209	1.980 257	0.105 370	0.000 725	0.002 899
2.00	0.806 375	0.839 125	1.939 086	0.152 800	−0.022 629	−0.090 516

Table 3. Coefficients for the γ functions in equations (3.6).

a	c_1	c_2	c_3	c_4	c_5	c_6	c_7
3.00	0.009 259	0.074 074	0.062 500	0.500 000	0.053 241	0.106 481	−0.092 593
2.90	0.013 240	0.091 345	0.052 617	0.459 230	0.039 378	0.091 971	−0.074 149
2.80	0.016 866	0.109 306	0.044 550	0.424 242	0.027 685	0.078 734	−0.055 764
2.70	0.020 366	0.128 207	0.037 746	0.394 061	0.017 380	0.066 463	−0.037 191
2.60	0.023 928	0.148 326	0.031 754	0.367 937	0.007 825	0.054 903	−0.018 174
2.50	0.027 720	0.169 970	0.026 158	0.345 280	−0.001 561	0.043 828	0.001 568
2.40	0.031 909	0.193 493	0.020 517	0.325 625	−0.011 391	0.033 033	0.022 341
2.30	0.036 679	0.219 306	0.014 273	0.308 596	−0.022 405	0.022 323	0.044 494
2.20	0.042 244	0.247 892	0.006 612	0.293 895	−0.035 632	0.011 501	0.068 425
2.10	0.048 869	0.279 832	−0.003 816	0.281 281	−0.052 685	0.000 362	0.094 608
2.00	0.056 893	0.315 823	−0.019 507	0.270 565	−0.076 400	−0.011 315	0.123 604

4. FPs and various types of critical behaviour

The nature of the critical behaviour is determined by the existence of a stable FP satisfying the system of equations

$$\beta_i(u^*, v^*, w^*) = 0 \qquad (i = 1, 2, 3). \tag{4.1}$$

It is well known that perturbation series are asymptotic series, and that the vertices describing the interaction of the order parameter fluctuations in the fluctuating region $m \to 0$ are large enough so that expressions (3.6) cannot be used directly. For this reason, to extract the required physical information from the obtained expressions, we employed the Padé–Borel approximation of the summation of asymptotic series extended to the multiparameter case [8]. The direct and inverse Borel transformations for the multiparameter case have the form

$$\begin{aligned} f(u, v, w) &= \sum_{i,j,k} c_{ijk} u^i v^j w^k = \int_0^\infty \mathrm{e}^{-t} F(ut, vt, wt)\,\mathrm{d}t \\ F(u, v, w) &= \sum_{i,j,k} \frac{c_{ijk}}{(i+j+k)!} u^i v^j w^k. \end{aligned} \tag{4.2}$$

A series in the auxiliary variable θ is introduced for analytical continuation of the Borel transform of the function:

$$\tilde{F}(u_1, u_2, u_3, \theta) = \sum_{k=0}^{\infty} \theta^k \sum_{i=0}^{k} \sum_{j=0}^{k-i} \frac{c_{i,j,k-i-j}}{k!} u_1^i u_2^j u_3^{k-i-j} \tag{4.3}$$

to which the $[L/M]$ Padé approximation is applied at the point $\theta = 1$. To perform the analytical continuation, the Padé approximant of $[L/1]$ type may be used which is known to provide rather good results for various Landau–Wilson models (see, e.g., [21, 22]). The property of preserving the symmetry of a system during application of the Padé approximation by the θ method, as in [21], has become important for multivertices models. We used the [2/1] approximant to calculate the β functions in the two-loop approximation.

However, the analysis of the series coefficients for the β_w function has shown that the summation of this series is fairly poor, which resulted in the absence of FP with $w^* \neq 0$, for example, in the case $m = 1$ for $a < 2.93$, in the case $m = 2$ for $a < 2.67$ etc. Dorogovtsev found the symmetry of the scaling function for the WH model in relation to the transformation $(u, v, w) \to (u, v, v + w)$ [9] which gives the possibility of investigating the problem of FP existence with $w^* \neq 0$ in the variables $(u, v, v+w)$. In this case our investigations have shown the existence of FPs with $w^* \neq 0$ in the whole region where the parameter a changes.

We have found three types of FPs in the physical region of parameter space $u^*, v^*, v^* + w^* \geqslant 0$ for different values of m and a. Type I corresponds to the FP of a pure system $(u^* \neq 0, v^*, w^* = 0)$, type II is a SR-disorder FP $(u^*, v^* \neq 0, w^* = 0)$ and type III corresponds to LR-disorder FPs $(u^*, v^*, w^* \neq 0)$. The type of critical behaviour of this disordered system for each value of m and a is determined by the stability of the corresponding FP. The requirement that the FP be stable reduces to the condition that the eigenvalues of the matrix

$$\boldsymbol{B}_{i,j} = \frac{\partial \beta_i(u_1^*, u_2^*, u_3^*)}{\partial u_j} \tag{4.4}$$

lie in the right-hand side complex half-plane.

Values of the stable FPs obtained for the most interesting values of the number of order-parameter components m and $2 \leqslant a \leqslant 3$ are presented in table 4. As one can see from this table, for the Ising model $(m = 1)$ the LR-disorder FP is stable for values of a in the whole investigated range. The additional calculations for $3 < a < 4$ have shown that only FP II is stable in this range. For $a = 3$ FP values for vertices u and $g(k)$ are equal, $u^* = 2.383\,38$ and $g^* = v^* + w^* = 0.551\,64$, and correspond to the SR-disordered Ising model FP, although $w^* \neq 0$. Similarly, for $m = 1$ and $a = 3$ the LR disorder is marginal, and the critical behaviour of the WH model, as that of the SR-disordered Ising model, is characterized by the same critical exponents (table 5). The critical behaviour of the XY-model $(m = 2)$ is determined by the LR-disorder FP for $a \leqslant 2.96$ and the SR-disorder FP for $a > 2.96$. The Heisenberg model $(m = 3)$ is characterized by a change in the types of critical behaviour from the LR-disorder type (III) for $a \leqslant 2.85$ to the pure type (I) for $a > 2.85$. Figure 1(*b*) shows regions of the various types of critical behaviour of the WH model, which we obtained in the two-loop approximation. The large change in the picture indicates that the correspondence between the WH results and our calculations in the two-loop approximation is weak.

However, the results, which we received for the disordered XY-model, must be corrected. We believe that in the higher field-theory orders of approximation k the critical behaviour of the XY-model will be determined by the FP of pure type (I) for $a_c^{(k)} < a$, but not by the SR-disorder FP (II), obtained in the two-loop order. Here, $a_c^{(k)}$ is a marginal value for a in the kth order of approximation, for which disorder is irrelevant ($a_c^{(6)} \simeq 2/\nu_0 = 2.99$ with $\nu_0 = 0.669$ [22] for

Table 4. Stable FPs of the 3D WH model from two-loop expansions.

	$m = 1$			$m = 2$			$m = 3$		
a	u^*	v^*	$w^* + v^*$	u^*	v^*	$w^* + v^*$	u^*	v^*	$w^* + v^*$
3.01	2.383 38	0.551 64	0.551 64	1.564 69	0.004 16	0.004 16	1.520 97	0.000 00	0.000 00
3.00	2.383 38	0.222 93	0.551 64	1.564 69	0.004 16	0.004 16	1.520 97	0.000 00	0.000 00
2.97	2.452 78	0.253 53	0.594 56	1.564 69	0.004 16	0.004 16	1.520 97	0.000 00	0.000 00
2.96	2.474 80	0.263 34	0.607 97	2.039 51	0.101 60	0.373 76	1.520 97	0.000 00	0.000 00
2.90	2.598 04	0.318 90	0.681 14	2.090 01	0.113 86	0.400 38	1.520 97	0.000 00	0.000 00
2.86	2.673 52	0.353 26	0.724 40	2.123 79	0.121 99	0.417 41	1.520 97	0.000 00	0.000 00
2.85	2.691 72	0.361 56	0.734 65	2.132 54	0.124 18	0.421 83	1.929 96	0.078 24	0.333 72
2.80	2.779 27	0.401 53	0.782 99	2.176 77	0.135 36	0.443 59	1.957 70	0.082 98	0.345 50
2.70	2.940 31	0.474 87	0.867 57	2.267 78	0.159 23	0.486 12	2.017 46	0.093 46	0.370 04
2.60	3.086 45	0.540 84	0.939 16	2.360 58	0.184 57	0.526 33	2.086 99	0.109 22	0.400 05
2.50	3.219 83	0.600 35	0.999 72	2.496 43	0.234 42	0.596 51	2.155 85	0.125 35	0.426 28
2.40	3.340 78	0.653 74	1.049 98	2.618 18	0.280 94	0.653 34	2.220 47	0.140 74	0.446 51
2.30	3.448 13	0.700 82	1.089 80	2.725 20	0.323 44	0.697 60	2.308 01	0.169 10	0.483 02
2.20	3.538 99	0.740 92	1.118 25	2.815 01	0.361 15	0.729 09	2.392 98	0.200 79	0.516 96
2.10	3.608 14	0.772 63	1.133 40	2.883 05	0.392 93	0.746 72	2.458 69	0.228 77	0.537 59
2.00	3.646 87	0.793 47	1.131 89	2.922 06	0.417 10	0.748 43	2.499 45	0.251 61	0.543 64

$m = 2$). Two facts indicate this, such as the weak stability of the SR-disorder FP revealed for $2.96 < a < 4$ and that $a_c^{(2)} = 3$ for $m_c = 2.0114$. In the higher orders of approximation the marginal value of m_c can be found with the use of the Harris criterion [2] $d\nu_0(m_c) - 2 = 0$, and as $\nu_0 = 0.669$ [23] for $m = 2$, then $m_c < 2$. Therefore, we believe that the corrected picture of the regions of various types of critical behaviour of the model with LR-correlated defects will be represented by figure 1(c).

It is obvious that for $m > 3$ the borderline equation between regions of pure and LR-disorder critical behaviour may be determined by the extended Harris criterion $a\nu_0(m) - 2 = 0$. The values of exponent ν and another static exponents for the pure 3D O(n)-symmetric model with $m > 3$ were obtained in the six-loop order by Antonenko and Sokolov in [21]. Therefore, marginal values of a_c for each $m > 3$ may be derived from these values ν_0 [21], so $a_c \simeq 2.71$ for $m = 4$, $a_c \simeq 2.61$ for $m = 5$, $a_c \simeq 2.53$ for $m = 6$ etc. Thus, as $m \to \infty$, $\nu_0 \to 1$ then $a_c \to 2$.

The case with $a = 2$ corresponds to a system of straight lines of impurities or straight dislocation lines of random orientation in a sample. The critical behaviour of the 3D model with lines of impurities of $s \gg 1$ uniformly distributed orientations was considered by Dorogovtsev in [23] in the one-loop order of approximation. This model is assumed to have similar critical properties to the WH model with $a = 2$. Dorogovtsev showed that the SR-disorder FP is stable for the number of order-parameter components $m < \frac{8}{5}$, and the LR-disorder FP is stable for $m > \frac{8}{5}$. Our description of the WH model in the two-loop approximation corrects Dorogovtsev's results and shows a low accuracy of the one-loop order considerations. The inaccuracy of Dorogovtsev's predictions can also be undersood with the use of the extended Harris criterion $a\nu_{SR}(m_c) - 2 = 0$ to determine the borderline equation between regions of SR- and LR-disorder behaviour. For $a = 2$ and marginal value m_c, ν_{SR} must be equal to one. But from table 5 we can see that $\nu_{SR}(m = 1) = 0.6715$ and $\nu_{SR}(m = 2) = 0.6642$ in the two-loop approximation and therefore $\nu_{SR} \neq 1$ for $1 \leqslant m \leqslant 2$.

Table 5. Critical exponents of the 3D WH model from two-loop expansions.

		$m = 1$			$m = 2$			$m = 3$		
a	$2/a$	η	ν	z	η	ν	z	η	ν	z
3.01		0.0327	0.6715	2.1712	0.0288	0.6642	2.0000	0.0283	0.6960	2.0217
3.00	0.6667	0.0327	0.6715	2.1712	0.0288	0.6642	2.0000	0.0283	0.6960	2.0217
2.90	0.6897	0.0304	0.6813	2.2120	0.0248	0.7141	2.1315	0.0283	0.6960	2.0217
2.80	0.7143	0.0270	0.6889	2.2486	0.0212	0.7190	2.1510	0.0179	0.7600	2.1128
2.70	0.7407	0.0227	0.6950	2.2837	0.0166	0.7240	2.1736	0.0137	0.7632	2.1269
2.60	0.7692	0.0176	0.7002	2.3184	0.0112	0.7692	2.1988	0.0084	0.7682	2.1443
2.50	0.8000	0.0118	0.7046	2.3532	0.0035	0.7378	2.2338	0.0025	0.7727	2.1633
2.40	0.8333	0.0055	0.7083	2.3879	−0.0050	0.7452	2.2684	−0.0040	0.7763	2.1827
2.30	0.8696	−0.0012	0.7114	2.4215	−0.0138	0.7513	2.3013	−0.0125	0.7835	2.2078
2.20	0.9091	−0.0081	0.7137	2.4524	−0.0226	0.7558	2.3301	−0.0218	0.7905	2.2315
2.10	0.9524	−0.0147	0.7151	2.4780	−0.0307	0.7588	2.3522	−0.0303	0.7952	2.2514
2.00	1.0000	−0.0205	0.7155	2.4949	−0.0371	0.7599	2.3649	−0.0370	0.7975	2.2644

5. Critical exponents and conclusions

Finally, we have calculated the static and dynamic critical exponents for the WH model (table 5), received from the resummed by the generalized Padé–Borel method γ functions in the corresponding stable FPs: $\eta = \gamma_\phi(u^*, v^*, w^*)$, $\nu = [2 + \gamma_{\phi^2}(u^*, v^*, w^*) - \gamma_\phi(u^*, v^*, w^*)]^{-1}$ and $z = 2 + \gamma_\lambda(u^*, v^*, w^*)$.

The comparison of the exponent ν values and ratio $2/a$ from table 5 shows the violation supposed in [1] on the basis of some heuristic arguments as exactly the relation $\nu = 2/a$. The revealed difference is caused by the use in our work of a more accurate field-theoretic description in the higher orders of approximation for the 3D system directly together with methods of series summation. Also, these distinctions can be explained by the application for calculations of the concrete numerical values of parameter a and taking into consideration the graphs of the form shown in figure 5, discarded when the ε, δ-expansion is used, but contributions of which are increased when the values a are removed from $a = 3$. Of course, there are errors in the present consideration determined by the accuracy of series summation for the β and γ functions. However, comparison of the exponent values for the SR-disorder Ising model, calculated with the use Padé–Borel method in [5, 6] in the two-loop and four-loop approximations respectively, shows that their differences are not more than 0.02. For the pure Ising model, comparison of the two-loop order results [5] with the high-order results [24] shows that differences of the exponent values are still smaller. At the same time, in our work ν–$2/a$ depends on the values of a and m and has the value 0.284, for example, for $a = 2$ and $m = 1$, which is considerably larger.

In [23, 25] Dorogovtsev calculated the static and dynamic critical exponents for a 3D model with a system of straight lines of impurities of $s \gg 1$ uniformly distributed orientations in the one-loop approximation. He found the exponent $z \simeq 2.41$ for $m = 2$ and $z \simeq 2.28$ for $m = 3$, and the exponent $\nu \simeq 0.75$, which is independent of m in the one-loop approximation. These values are comparable with the values of the same exponents in table 5 for the case $a = 2$. It should be emphasized that the value of exponent ν, obtained by Dorogovtsev, also demonstrates the violation of relation $\nu = 2/a$.

We also estimated the values of exponents η and z, derived for the WH model in [11] with the use of double ε, δ-expansion, by direct substitution of $\varepsilon = 1$ and $\delta = 2$ ($a = 2$). So, $\eta \simeq -0.071$ and $z \simeq 2.737$ for $m = 2$, $\eta \simeq -0.047$ and $z \simeq 2.494$ for $m = 3$. The resummation by the Padé–Borel method of second-order ε, δ-expansion series for z gave the values $z \simeq 2.566$ for $m = 2$ and $z \simeq 2.525$ for $m = 2$. The comparison of these values with values of the same exponents from table 5 shows that results obtained by field-theoretic description of the 3D WH model for the concrete numerical values of parameter a essentially differ from results evaluated by ε, δ-expansion.

In closing, we hope that the features of the critical behaviour of the WH model revealed in our paper will stimulate the organization of experimental works in real disordered systems with LR-correlated defects such as magnetic materials with line defects and solids with fractal-like defects. Also, computational methods can be applied to simulate disordered systems with straight lines of impurities of random orientation in a sample ($a = 2$). The received values of exponents can be used for an explanation of the results of a computer simulation of the 3D disordered Ising model [26] at impurity concentrations between the threshold of impurity percolation and the spin-percolation threshold, in which the fractal-like behaviour of impurity-extended structures and the competition between impurity-percolating and spin-percolating clusters are possible.

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